Gluon Structure of Hadrons and Nuclei

Phiala Shanahan
The College of William and Mary
Thomas Jefferson National Accelerator Facility
Gluon Structure

- Past 60+ years: detailed view of quark structure of nucleons
- Gluonic structure (beyond gluon density) relatively unexplored

**Gluon Structure of N* spectrum**
- Better understand and classify N* resonances
- Identify gluonic excitations

- Electron-Ion Collider
  - Priority in 2015 nuclear physics long range plan
  - “Understanding the glue that binds us all”
- Insights from Lattice QCD?
Gluonic Structure

Studying gluonic structure of hadrons/nuclei is hard

- Gluon probed only indirectly in electron scattering from hadrons/nuclei (does not couple to photon)
- Other processes less clean: heavy flavour production
- Quarks and gluons mix via evolution
  - **Uniquely quarky:** nonsinglet quantities
  - **Uniquely gluonic:** double helicity flip/ gluonic transversity
Leading twist gluon parton distribution $\Delta(x,Q^2)$: double helicity flip [Jaffe & Manohar 1989]

- Unambiguously gluonic: no analogous quark PDF at twist-2
- Non-vanishing in forward limit for targets with spin $\geq 1$
- Experimentally measurable in unpolarised electron DIS on polarised target
  - Nitrogen target: JLab LoI 2015
  - Polarised nuclei at EIC
- Moments calculable in LQCD
Double helicity flip structure function $\Delta(x, Q^2)$

Changes both photon and target helicity by 2 units
Gluonic Transversity

Double helicity flip structure function $\Delta(x,Q^2)$

- **Hadrons:** Gluonic Transversity (parton model interpretation)

$$\Delta(x, Q^2) = -\frac{\alpha_s(Q^2)}{2\pi} \text{Tr} Q^2 x^2 \int_x^1 \frac{dy}{y^3} \left[g_{\hat{x}}(y, Q^2) - g_{\hat{y}}(x, Q^2)\right]$$

$g_{\hat{x},\hat{y}}(y, Q^2)$: probability of finding a gluon with momentum fraction $y$ linearly polarised in $\hat{x}, \hat{y}$ direction

- **Nuclei:** Exotic Glue

  gluons not associated with individual nucleons in nucleus

  $\langle p | \mathcal{O} | p \rangle = 0$

  $\langle N, Z | \mathcal{O} | N, Z \rangle \neq 0$
Gluonic Transversity

Double helicity flip structure function $\Delta(x, Q^2)$

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\]

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- **Nuclei:** Exotic Glue

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\[
\langle p|\mathcal{O}|p \rangle = 0
\]

\[
\langle N, Z|\mathcal{O}|N, Z \rangle \neq 0
\]
Moments of $\Delta(x, Q^2)$ are calculable in LQCD

\[
\int_0^1 dx x^{n-1} \Delta(x, Q^2) = \frac{\alpha_s(Q^2)}{3\pi} \frac{A_n(Q^2)}{n + 2}, \quad n = 2, 4, 6 \ldots ,
\]

Determined by matrix elements of local gluonic operators

\[
\langle pE' | S \left[ G_{\mu_1} \overleftrightarrow{D}_{\mu_3} \ldots \overleftrightarrow{D}_{\mu_n} G_{\nu_2} \right] | pE \rangle = (-2i)^{n-2} S \left[ (p_{\mu} E'_{\mu_1} - p_{\mu_1} E'_{\mu}) (p_{\nu} E_{\mu_2} - p_{\mu_2} E_{\nu}) + (\mu \leftrightarrow \nu) \right] p_{\mu_3} \ldots p_{\mu_n} A_n(Q^2) \ldots ,
\]

Symmetrise in $\mu_1, \ldots, \mu_n$, trace subtract in all free indices
Simplest spin-1 system: $\phi$ meson (unphysically heavy)

Different basis vectors

Different irreps.

Boost of $\phi$ (not mtm transfer)

W. Detmold, PES, PRD 94 (2016), 014507
Spin-indep. gluon structure

W. Detmold, PES, PRD 94 (2016), 014507

Spin-independent gluon operator:

\[
\overline{O}_{\mu_1 \ldots \mu_n} = S \left[ G_{\mu_1 \alpha} \overleftrightarrow{D}_{\mu_3} \ldots \overleftrightarrow{D}_{\mu_n} G_{\mu_2}^\alpha \right]
\]

Matrix elements at \( n=2 \) define lowest moment of structure functions

\[
\langle pE'| \overline{O}_{\mu_1 \mu_2} | pE \rangle = S \left[ M^2 E'_{\mu_1} E_{\mu_2} \right] B_{2,1}(\mu^2) + S \left[ (E \cdot E'^*) p_{\mu_1} p_{\mu_2} \right] B_{2,2}(\mu^2)
\]

Two reduced matrix elements

- Analysis as in transversity case
- Mixing with quark ops. neglected, pQCD calcs. shown that it is small: Alexandrou 1611.06901
How does the gluon radius of a proton compare to the quark/charge radius?

- **Bag Model**: gluon radius > charge radius
- **Constituent Quark Model**: gluon radius ~ charge radius
- **LQCD with heavy quarks**: gluon radius < charge radius

Or is the picture more complicated?
Gluon Generalised FFs

Matrix elements of the spin-independent gluon structure function

- Off-forward matrix elements are complicated:

\[
\langle p' E' | S \left[ G_{\mu\alpha} i \overleftrightarrow{D}_{\mu_1} \cdots i \overleftrightarrow{D}_{\mu_n} G^{\alpha} \right] | p E \rangle 
= \sum_{\text{m even}}^{n} \left\{ B_{1,m}^{(n+2)} (\Delta^2) M^2 S \left[ E_\mu E'_{\nu} \Delta_{\mu_1} \cdots \Delta_{\mu_m} P_{\mu_{m+1}} \cdots P_{\mu_n} \right] 
+ B_{2,m}^{(n+2)} (\Delta^2) S \left[ (E \cdot E') P_{\mu} P_{\nu} \Delta_{\mu_1} \cdots \Delta_{\mu_m} P_{\mu_{m+1}} \cdots P_{\mu_n} \right] 
+ B_{3,m}^{(n+2)} (\Delta^2) S \left[ (E \cdot E') \Delta_{\mu} \Delta_{\nu} \Delta_{\mu_1} \cdots \Delta_{\mu_m} P_{\mu_{m+1}} \cdots P_{\mu_n} \right] 
+ B_{4,m}^{(n+2)} (\Delta^2) S \left[ ((E' \cdot P) E_{\mu} P_{\nu} + (E \cdot P) E'_{\mu} P_{\nu}) \Delta_{\mu_1} \cdots \Delta_{\mu_m} P_{\mu_{m+1}} \cdots P_{\mu_n} \right] 
+ B_{5,m}^{(n+2)} (\Delta^2) S \left[ ((E' \cdot P) E_{\mu} \Delta_{\nu} - (E \cdot P) E'_{\mu} \Delta_{\nu}) \Delta_{\mu_1} \cdots \Delta_{\mu_m} P_{\mu_{m+1}} \cdots P_{\mu_n} \right] 
+ \frac{B_{6,m}^{(n+2)} (\Delta^2)}{M^2} \left[ (E \cdot P) (E' \cdot P) P_{\mu} P_{\nu} \Delta_{\mu_1} \cdots \Delta_{\mu_m} P_{\mu_{m+1}} \cdots P_{\mu_n} \right] 
+ \frac{B_{7,m}^{(n+2)} (\Delta^2)}{M^2} \left[ (E \cdot P) (E' \cdot P) \Delta_{\mu} \Delta_{\nu} \Delta_{\mu_1} \cdots \Delta_{\mu_m} P_{\mu_{m+1}} \cdots P_{\mu_n} \right] \right\}.
\]
Matrix elements of the spin-independent gluon structure function

Off-forward matrix elements are complicated:

\[
\begin{aligned}
\langle p'E' | S \left[ G_{\mu\alpha} i \overleftrightarrow{D}_{\mu_1} \ldots i \overleftrightarrow{D}_{\mu_n} G_{\nu}^\alpha \right] | pE \rangle \\
= \sum_{m\text{ even}}^{n} \left\{ \frac{B_{1,m}^{(n+2)}(\Delta^2)}{M^2} M^2 S [ E_\mu E_\nu^* \Delta_{\mu_1} \ldots \Delta_{\mu_m} P_{\mu_{m+1}} \ldots P_{\mu_n}] \\
+ \frac{B_{2,m}^{(n+2)}(\Delta^2)}{M^2} M^2 S [ (E \cdot E^*) P \cdot P \Delta_{\mu_1} \ldots \Delta_{\mu_m} P_{\mu_{m+1}} \ldots P_{\mu_n}] \\
+ \frac{B_{3,m}^{(n+2)}(\Delta^2)}{M^2} M^2 S [ (E \cdot E^*) (E' \cdot P) P_{\mu_1} P_{\nu} \Delta_{\mu_1} \ldots \Delta_{\mu_m} P_{\mu_{m+1}} \ldots P_{\mu_n}] \\
+ \frac{B_{4,m}^{(n+2)}(\Delta^2)}{M^2} M^2 S [ (E \cdot E^*) (E' \cdot P) (E' \cdot P) P_{\mu_1} P_{\nu} \Delta_{\mu_1} \ldots \Delta_{\mu_m} P_{\mu_{m+1}} \ldots P_{\mu_n}] \\
+ \frac{B_{5,m}^{(n+2)}(\Delta^2)}{M^2} M^2 S [ (E \cdot E^*) (E' \cdot P) (E' \cdot P) (E' \cdot P) P_{\mu_1} P_{\nu} \Delta_{\mu_1} \ldots \Delta_{\mu_m} P_{\mu_{m+1}} \ldots P_{\mu_n}] \\
+ \frac{B_{6,m}^{(n+2)}(\Delta^2)}{M^2} M^2 S [ (E \cdot E^*) (E' \cdot P) (E' \cdot P) (E' \cdot P) (E' \cdot P) P_{\mu_1} P_{\nu} \Delta_{\mu_1} \ldots \Delta_{\mu_m} P_{\mu_{m+1}} \ldots P_{\mu_n}] \\
+ \frac{B_{7,m}^{(n+2)}(\Delta^2)}{M^2} M^2 S [ (E \cdot E^*) (E' \cdot P) (E' \cdot P) (E' \cdot P) (E' \cdot P) (E' \cdot P) P_{\mu_1} P_{\nu} \Delta_{\mu_1} \ldots \Delta_{\mu_m} P_{\mu_{m+1}} \ldots P_{\mu_n}] \right\}.
\end{aligned}
\]

Many gluonic radii:
Defined by slope of each form factor at \( Q^2 = t = 0 \)
One GFF can be resolved for all momenta

Different H(4) irreps

Spin-Indep. Gluon GFFs

Three GFFs can be resolved for all momenta

Quark and Gluon GFFs

Ratio of gluon to quark unpolarised GFFs

Gluon vs quark radius is a non-trivial question
Much more complicated than intuitive pictures
First investigations:
\( \phi \) meson
simplest spin-1 system (has fwd limit gluon transversity)

Phenomenologically relevant:
nucleon, \( N^* \), excited mesons, nuclei
Gluon structure - nuclei

**European Muon Collaboration (1983):**

Modification of per-nucleon cross section of nucleons bound in nuclei

Precise understanding of nuclear targets essential for DUNE expt: extraction of neutrino mass hierarchy, mixing parameters

**Ratio of structure function $F_2$ per nucleon for iron and deuterium**

$$F_2(x, Q^2) = \sum_{q=u,d,s...} xz_q^2 [q(x, Q^2) + \bar{q}(x, Q^2)]$$

What is the gluonic analogue of the EMC effect?
Gluon momentum fraction

NPLQCD Collaboration, in preparation

- Spin-independent structure function in nucleon and light nuclei
- Present statistics: can’t distinguish from no-EMC effect scenario
- Small additional uncertainty from mixing with quark operators

\[ m_\pi \sim 450 \text{ MeV} \quad \text{and} \quad m_\pi \sim 800 \text{ MeV} \]
Gluon structure circa 2025

- Electron-ion collider will dramatically alter our knowledge of the gluonic structure of hadrons and nuclei
- Work towards a complete 3D picture of parton structure (moments, x-dependence of PDFs, GPDs, TMDs)
- $\Delta(x,Q^2)$ has an interesting role
  - Purely gluonic
  - Non-nucleonic: directly probe nuclear effects
- Compare quark and gluon distributions in hadrons and nuclei
- Potentially reveal new information about the nature of gluon excitations and the $N^*$ spectrum
- Lattice QCD calculations in hadrons and light nuclei will complement and extend understanding of fundamental structure of nature