Finite-Energy Sum Rules:

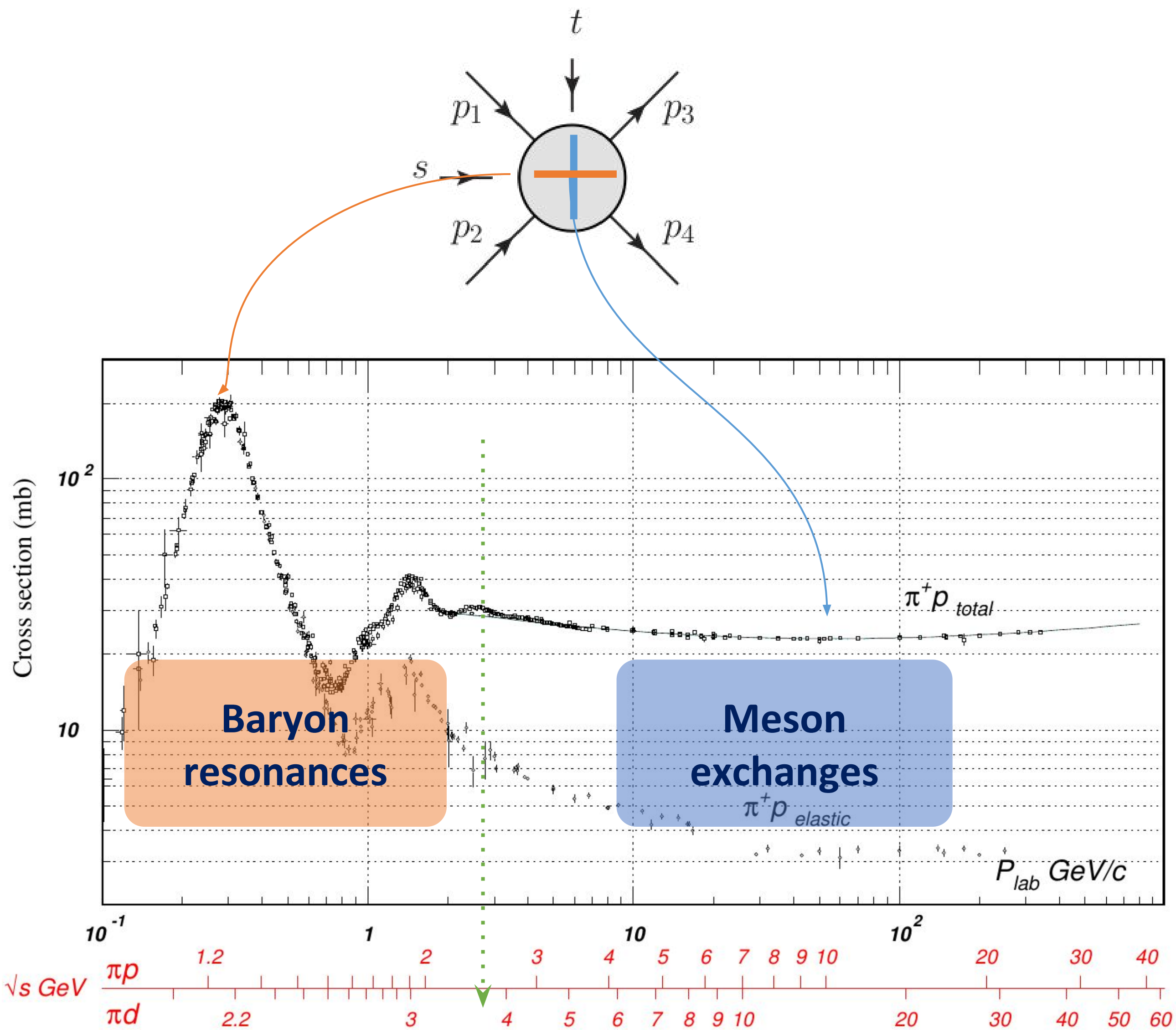
Going high to solve low-energy issues

Jannes Nys

JPAC Collaboration

NSTAR2017, Columbia, South Carolina (August 20 - 23, 2017)

Overview



Connect low- and high-energy dynamics.

Formalism

$$A_{\lambda';\lambda\,\lambda_\gamma}(s,t) = \overline{u}_{\lambda'}(p')\left(\sum_{k=1}^{4} A_k(s,t) M_k\right) u_\lambda(p)$$

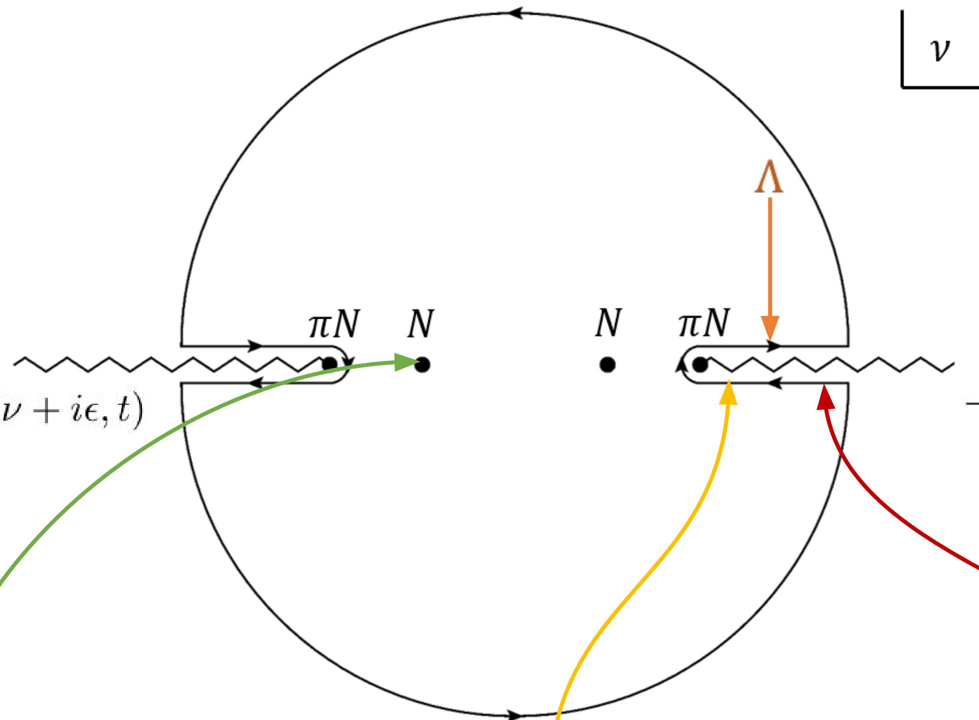
- No kinematic singularities
- No kinematic zeros
- Discontinuities:
  - Unitarity cut
  - Nucleon pole

$$M_k \equiv M_k(s,t,\lambda_\gamma)$$

$$M_1 = \frac{1}{2}\gamma_5\gamma_\mu\gamma_\nu F^{\mu\nu}\,,$$
$$M_2 = 2\gamma_5 q_\mu P_\nu F^{\mu\nu}\,,$$
$$M_3 = \gamma_5\gamma_\mu q_\nu F^{\mu\nu}\,,$$
$$M_4 = \frac{i}{2}\epsilon_{\alpha\beta\mu\nu}\gamma^\alpha q^\beta F^{\mu\nu}\,.$$

Dispersion relations - FESR

$$A_i(\nu,t)\nu^k$$



Fixed t

Crossing variable: $\nu = \frac{s-u}{4\,M}$

s-channel: $+\,\nu$

u-channel: $-\,\nu$

$$A_i^\sigma(-\nu - i\epsilon, t) = \xi_i A_i^\sigma(\nu + i\epsilon, t)$$

$$-\beta_i(t)\frac{\tau + e^{-i\pi\alpha(t)}}{\sin \pi\alpha(t)}(r_i\nu)^{\alpha(t)-1}$$

$$\pi B_i^\sigma(t)\left(\frac{\nu_M}{\Lambda}\right)^k + \int_{\nu_\pi}^{\Lambda} \mathrm{Im}\, A_i^\sigma(\nu',t)\left(\frac{\nu'}{\Lambda}\right)^k d\nu' = \beta_i^\sigma(t)\frac{(\Lambda/\nu_0)^{\alpha(t)}}{\alpha(t)+k}\nu_0$$

Nucleon pole | Low-energy model | Regge pole

Analyticity results in Finite-Energy Sum Rules.

Low energies

$$\int_{\nu_\pi}^{\Lambda} \mathrm{Im}\ A_i^{\sigma}(\nu', t) \left(\frac{\nu'}{\Lambda}\right)^k d\nu'$$

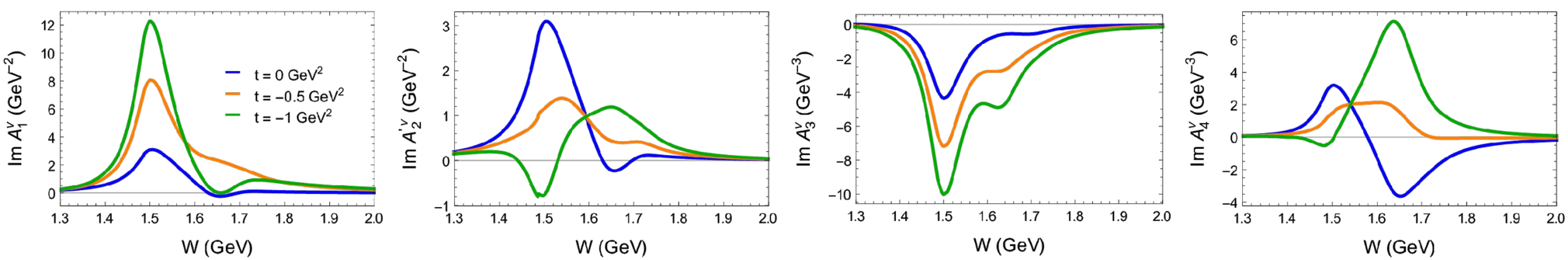
Low energy models

- BnGa, Julich-Bonn, ANL-Osaka, SAID, MAID
- Eta-MAID 2001 **[Chiang et al., Nucl. Phys. A 700, 429 (2002)]**
  - Isospin decomposed amplitudes:

$$A_i^{ab} = A_i^s \delta^{ab} + A_i^v \tau_3^{ab}$$

$$A_i^p = A_i(\gamma p \to \eta p) = A_i^s + A_i^v$$

$$A_i^n = A_i(\gamma n \to \eta n) = A_i^s - A_i^v$$

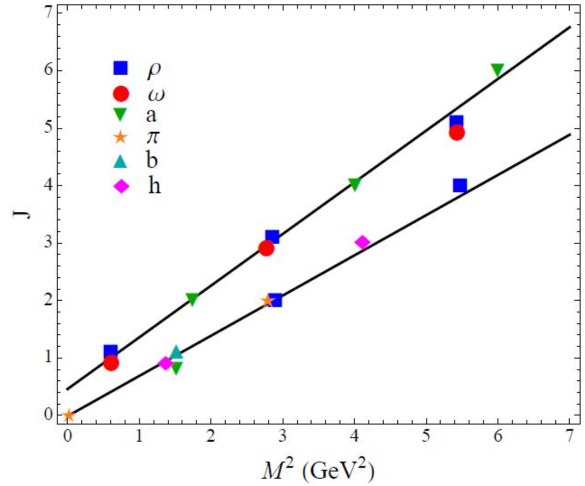


High energies

Regge pole model

$$J(m^2)$$

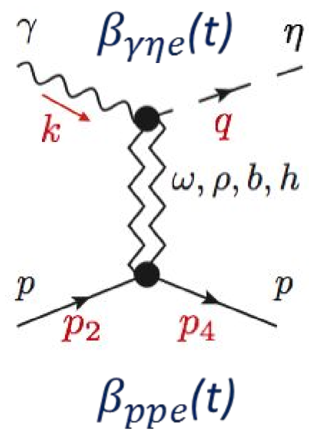
$$A_{i,R}(\nu,t) = \beta_i(t)\frac{\tau + e^{-i\pi\alpha(t)}}{\sin\pi\alpha(t)}(r_i\nu)^{\alpha(t)-1}$$



Dominant: vector exchanges

A_i	I^G	J^{PC}	η	Leading exchanges
A_1	$0^-, 1^+$	$(1,3,5,...)^{--}$	$+1$	$\rho(770), \omega(782)$
A_2'	$0^-, 1^+$	$(1,3,5,...)^{+-}$	-1	$h_1(1170), b_1(1235)$
A_3	$0^-, 1^+$	$(2,4,...)^{--}$	-1	$\rho_2(??), \omega_2(??)$
A_4	$0^-, 1^+$	$(1,3,5,...)^{--}$	$+1$	$\rho(770), \omega(782)$

$$A_2' = A_1 + tA_2$$



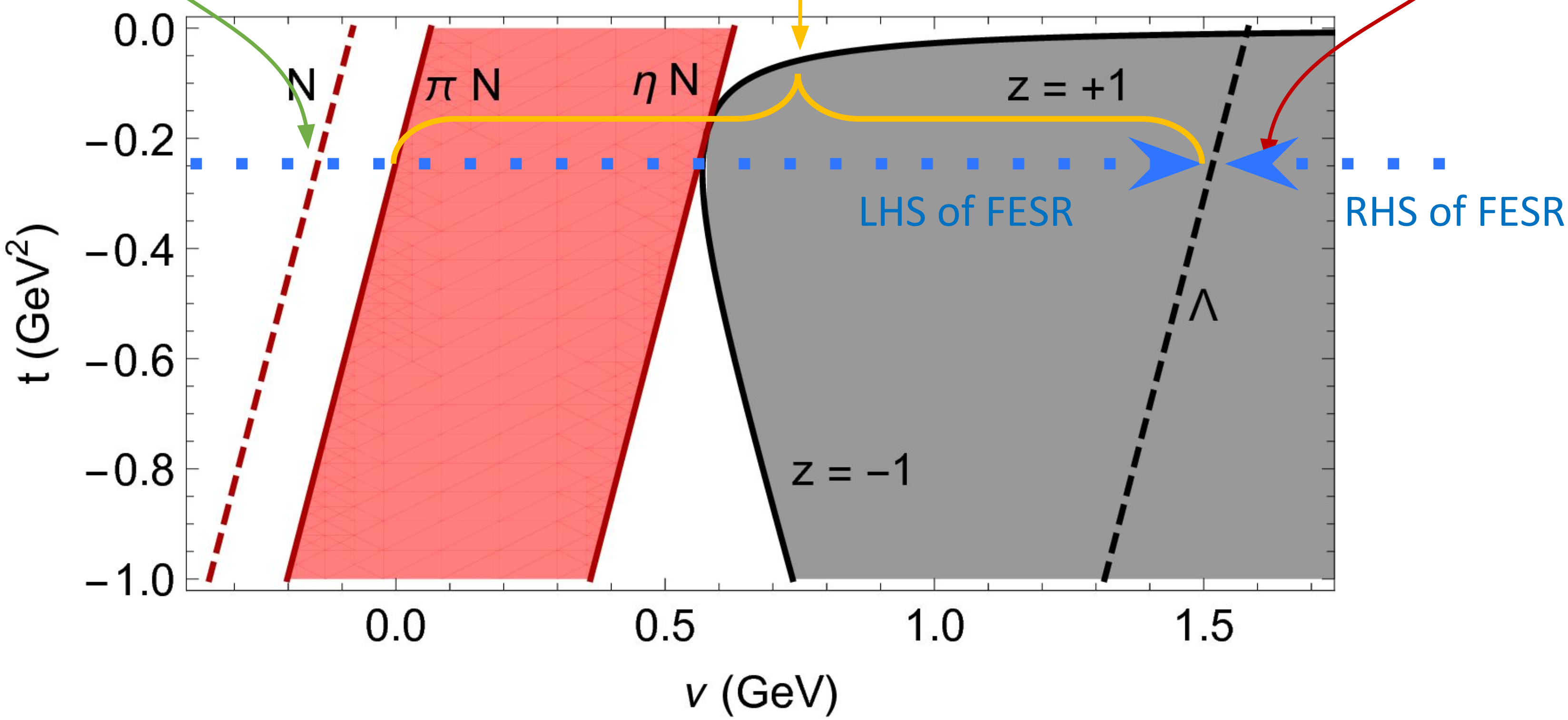
$$\gamma p \to \eta p\,, \qquad A = (\omega + h + \omega_2) + (\rho + b + \rho_2)$$

$$\gamma n \to \eta n\,, \qquad A = (\omega + h + \omega_2) - (\rho + b + \rho_2)$$

Dispersion relations

$$\pi B_i^\sigma(t)\left(\frac{\nu_M}{\Lambda}\right)^k + \int_{\nu_\pi}^{\Lambda} \mathrm{Im}\, A_i^\sigma(\nu',t)\left(\frac{\nu'}{\Lambda}\right)^k d\nu' = \beta_i^\sigma(t)\frac{(\Lambda/\nu_0)^{\alpha(t)}}{\alpha(t)+k}\nu_0$$

Nucleon pole — Low-energy model — Regge pole



Matching: natural exchanges

$$\pi B_i^\sigma(t)\left(\frac{\nu_M}{\Lambda}\right)^k + \int_{\nu_\pi}^{\Lambda} \mathrm{Im}\, A_i^\sigma(\nu',t)\left(\frac{\nu'}{\Lambda}\right)^k d\nu' = \beta_i^\sigma(t)\frac{(\Lambda/\nu_0)^{\alpha(t)}}{\alpha(t)+k}\nu_0$$

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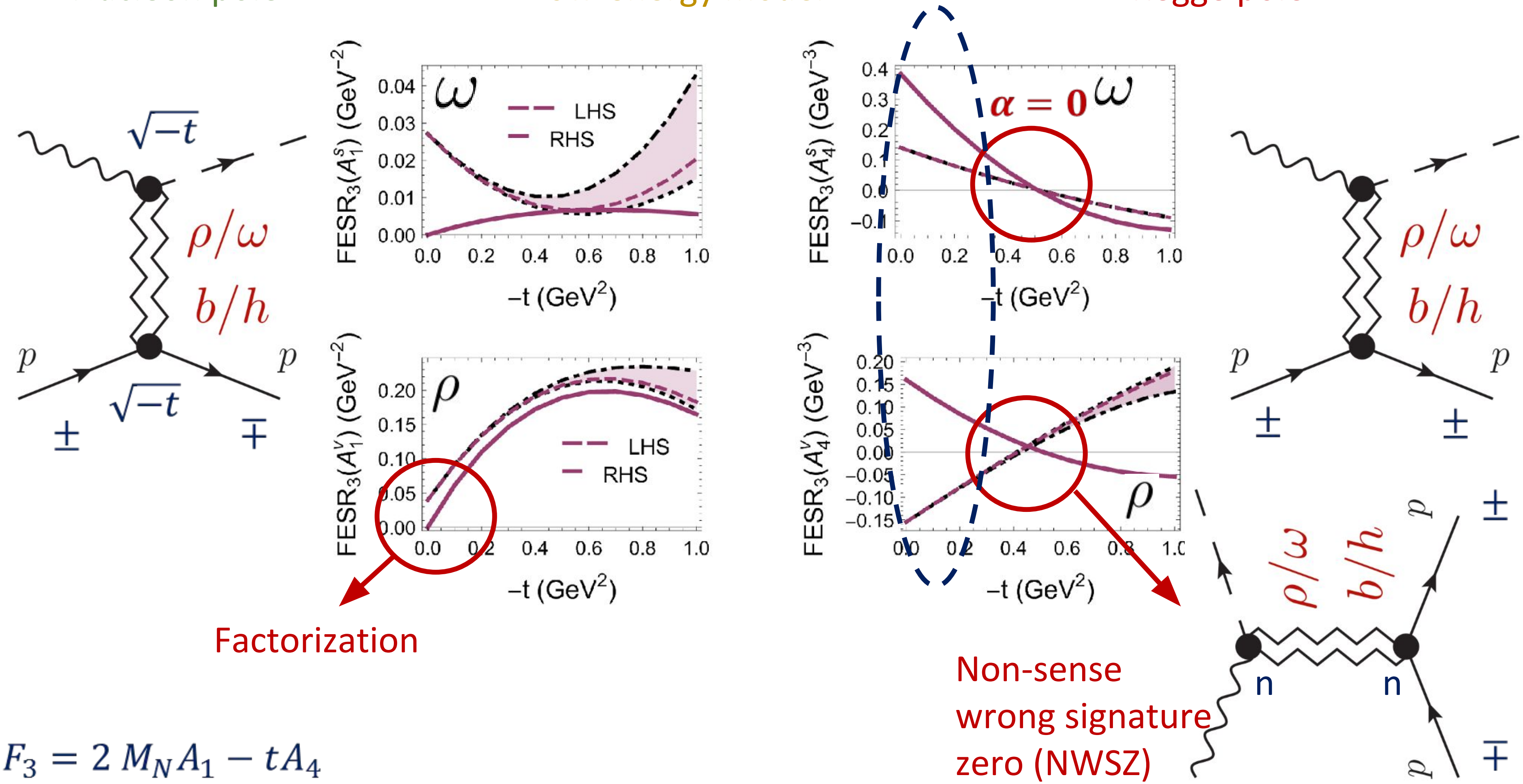
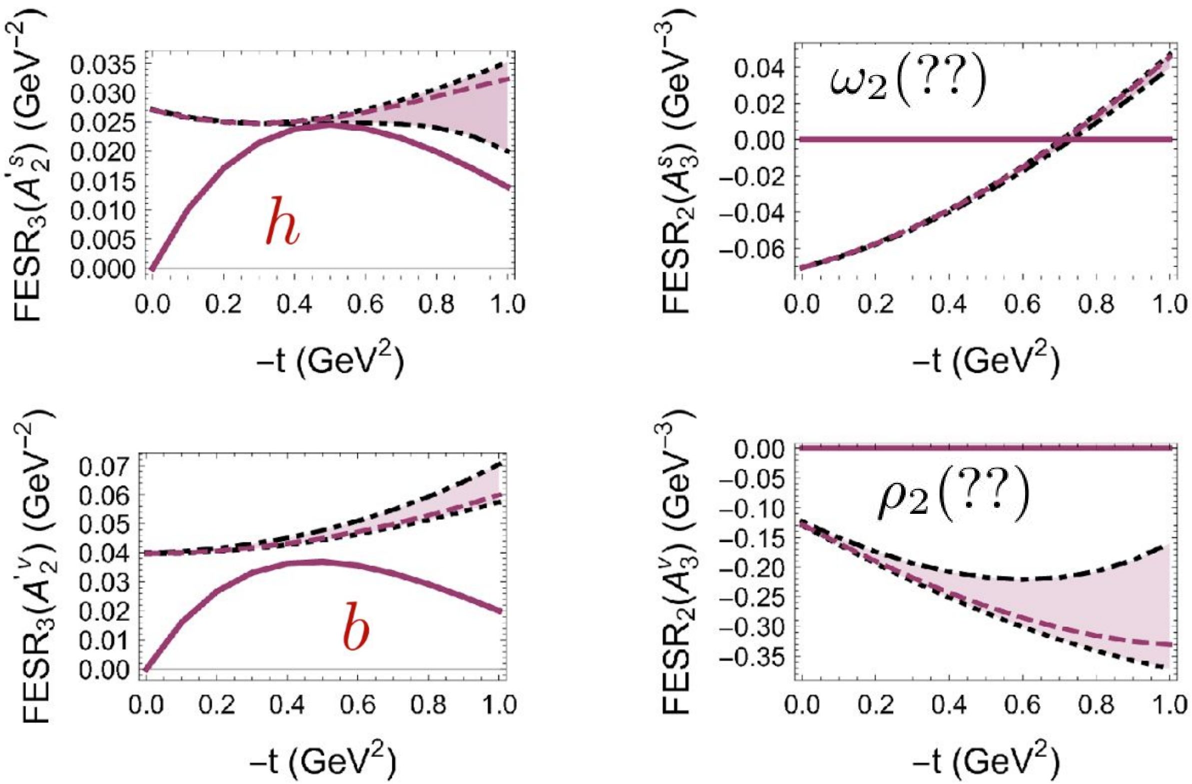


$F_3 = 2\, M_N A_1 - t A_4$

Matching: unnatural exchanges

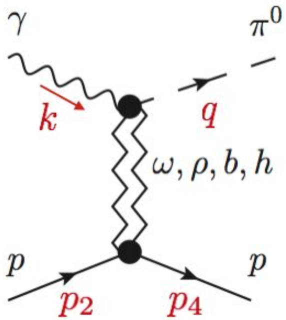
$$\pi B_i^\sigma(t)\left(\frac{\nu_M}{\Lambda}\right)^k + \int_{\nu_\pi}^{\Lambda} \mathrm{Im}\, A_i^\sigma(\nu',t)\left(\frac{\nu'}{\Lambda}\right)^k d\nu' = \beta_i^\sigma(t)\frac{(\Lambda/\nu_0)^{\alpha(t)}}{\alpha(t)+k}\nu_0$$

Nucleon pole — Low-energy model — Regge pole

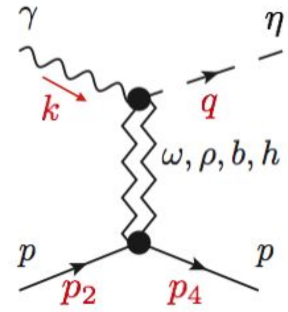


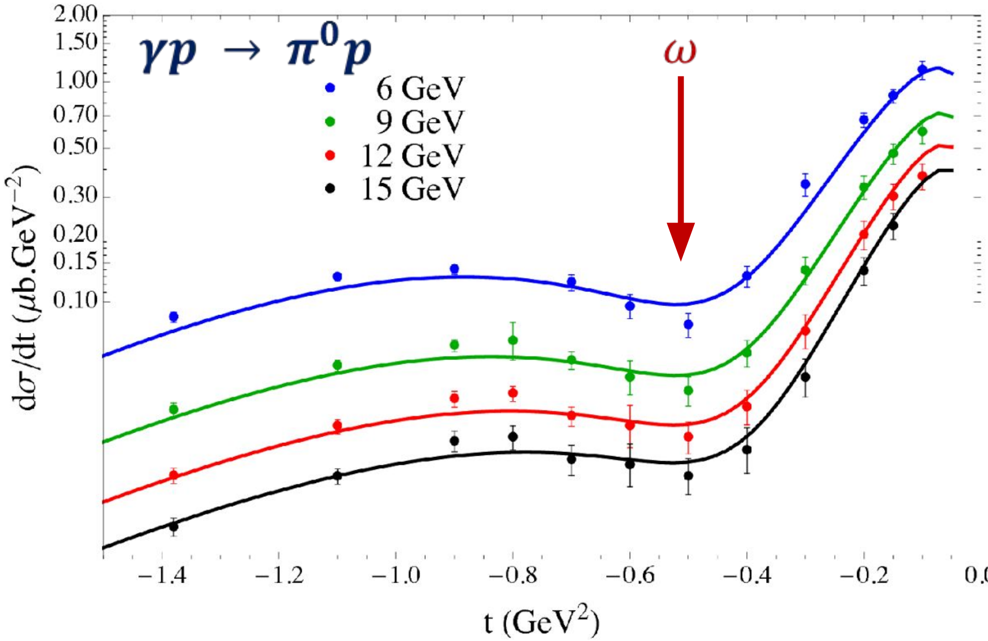
Look for unnatural contributions in the beam asymmetry

Data

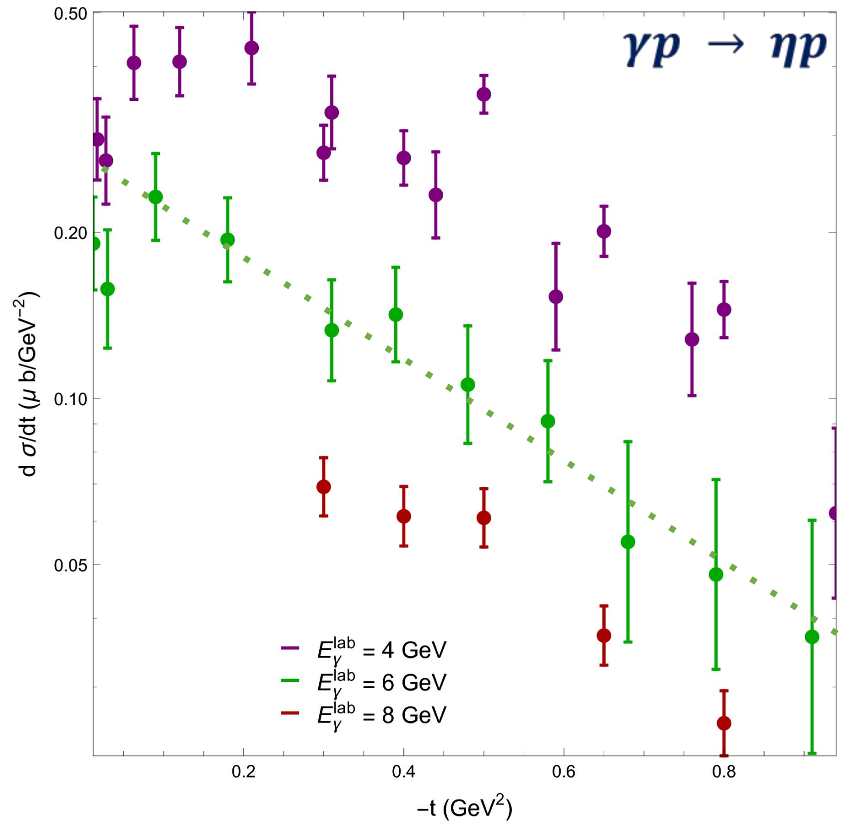


$$A(\eta) = \sqrt{3}\mathcal{A}\left[A_\rho(\pi^0) + A_b(\pi^0) + \frac{1}{9}\left(A_\omega(\pi^0) + A_h(\pi^0)\right)\right]$$



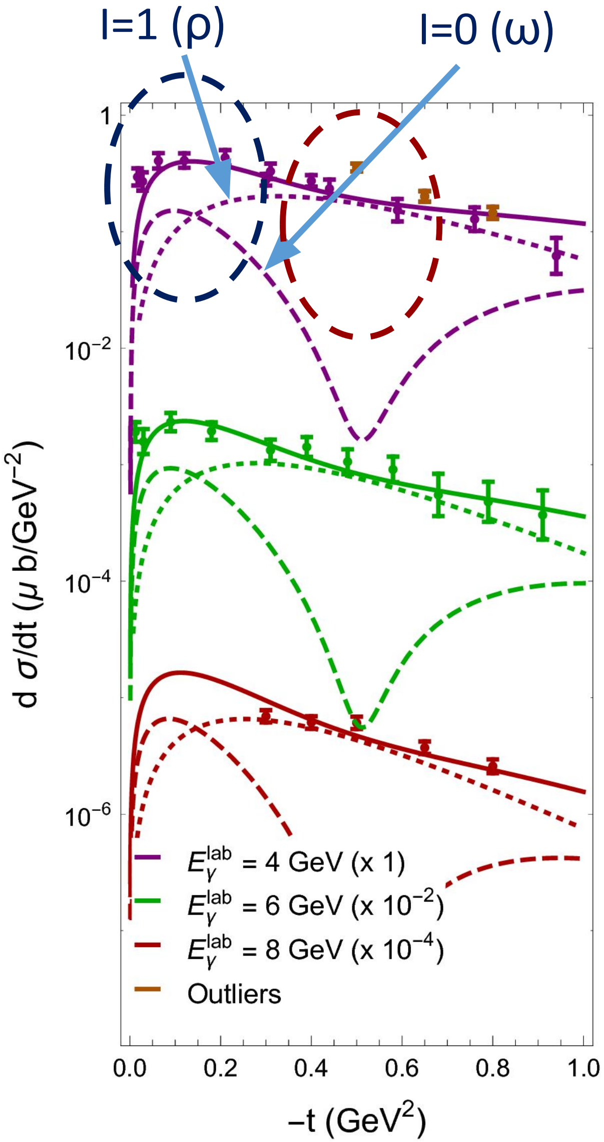


[V. Mathieu]



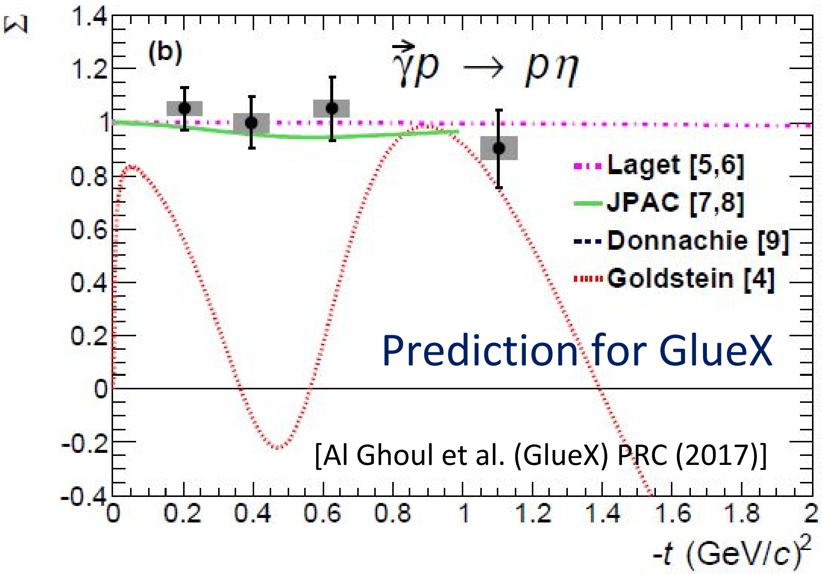
[Data: Dewire 1971, Braunschweig 1970]

Results



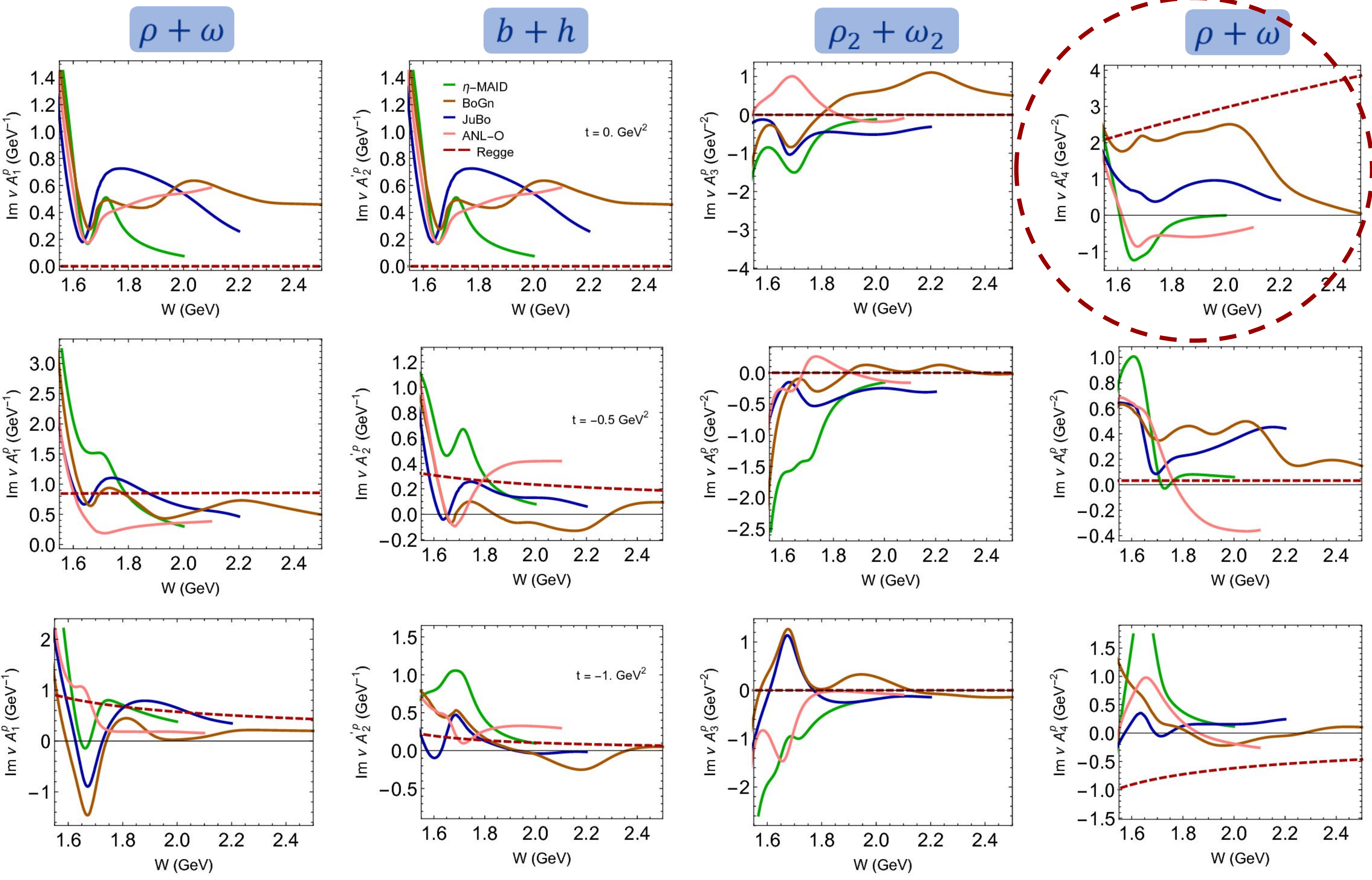
Natural dominant: $\Sigma = +1$

Unnatural dominant: $\Sigma = -1$



Fill up the dip with natural contribution: ρ

Comparison for $\gamma p \rightarrow \eta p$



η'/η beam asymmetry

$$\Sigma^{(\prime)} = \frac{d\sigma_\perp^{(\prime)} - d\sigma_\parallel^{(\prime)}}{d\sigma_\perp^{(\prime)} + d\sigma_\parallel^{(\prime)}}$$

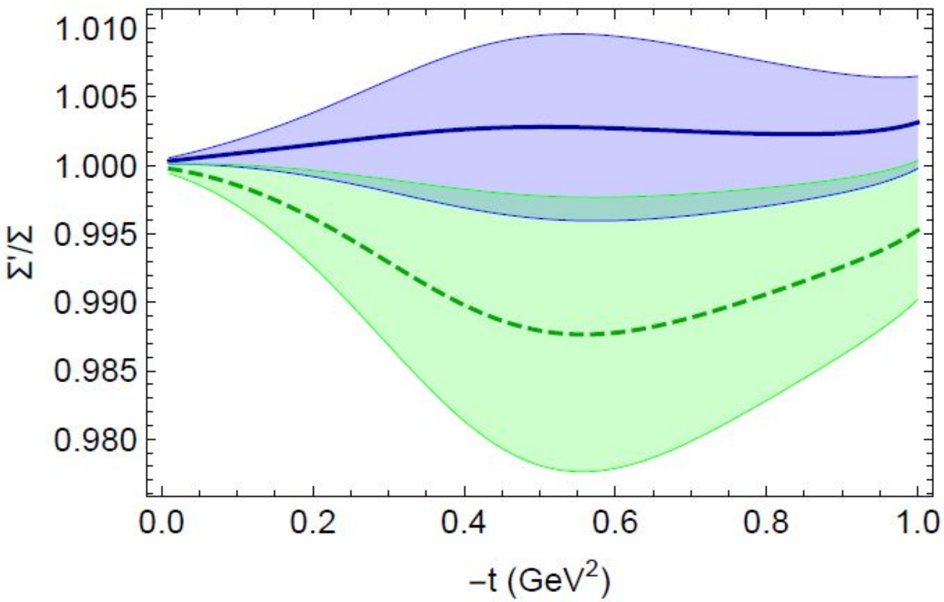
$$\frac{\Sigma'}{\Sigma} = 1 + \frac{1-\Sigma^2}{\Sigma} \cdot \frac{k_V - k_A}{(1+\Sigma)k_V + (1-\Sigma)k_A}$$

$$k_V = \frac{d\sigma'_\perp}{d\sigma_\perp}, \qquad k_A = \frac{d\sigma'_\parallel}{d\sigma_\parallel}.$$

Quark model predictions:

$$k_V = k_A = \tan^2 \varphi$$

V.Mathieu, J.N. *et al.* (JPAC) [arXiv:1704.07684]



Dominant exchanges: ρ, ω

Variations: b, h radiative decays

Sizable deviation from 1:

- Non-negligible contributions from **hidden strangeness**
- Signicant deviation from the quark model description

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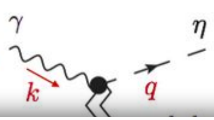
This project is supported by NSF

$\gamma p \to \eta p$

We present the model published in [Nys16] .
The differential cross section for $\gamma p \to \eta p$ is computed with Regge amplitudes in the domain $E_\gamma \geq 4$ GeV and $0 \leq -t \leq 1$ (in GeV^2).
We use the CGLN invariant amplitudes A_i defined in [Chew57a].
See the section Formalism for the definition of the variables.
The model and its context is detailed in [Nys16] . We report here only th
features of the model.



Formalism

The differential cross section is a function of 2 kinematic variables. The f
laboratory frame E_γ (in GeV) or the total energy squared s (in GeV^2). T
scattering angle in the rest frame $\cos\theta$ or the momentum transfered squar
The momenta of the particles are k (photon), q (eta), p_2 (target) and p_4 (r
μ and the proton mass is M_N. The Mandelstam variables, $s = (k + p_2)^2$,
are related through $s + t + u = 2M_N^2 + \mu^2$. Furthermore, we intro

- **Publication:** [Nys16]
- **C/C++ observables:** C-code main, Input file, C-code source, C-code header, Eta-MAID 2001 multip
- **C/C++ minimal script to calculate the amplitudes:** C-code zip
- **Data:** Dewire , Braunschweig
- **Contact person:** Jannes Nys
- **Last update:** November 2016

Step-by-step introduction to calculating the model amplitudes of the high-energy model.

[hide] [show]

Run the code

Choose the beam energy in the lab frame E_γ, the other variable (t or $\cos\theta$) and its minimal, maximal, and increment values.
If you choose t (cos) only the min, max and step values of t ($\cos\theta$) are read.
Only physical t-values are calculated. Hence, for example $t = 0$ will be set to $t(\cos\theta = +1)$. Below $W = 2$ GeV, we use the Eta-MAID 2001 model using the lowest $l \leq 5$ multipoles. Above $W = 2$ GeV, the Regge model is evaluated. There is no smooth transition.

E_γ in GeV 9

t cos

t in GeV^2 (min max step) -1 0 0.01

$\cos\theta$ (min max step) 0.85 1 0.01

Start reset

November 2016:

- The $\gamma p \to \eta p$ page is online.

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Summary

[J.N., et al. (JPAC) PRD (2017)]

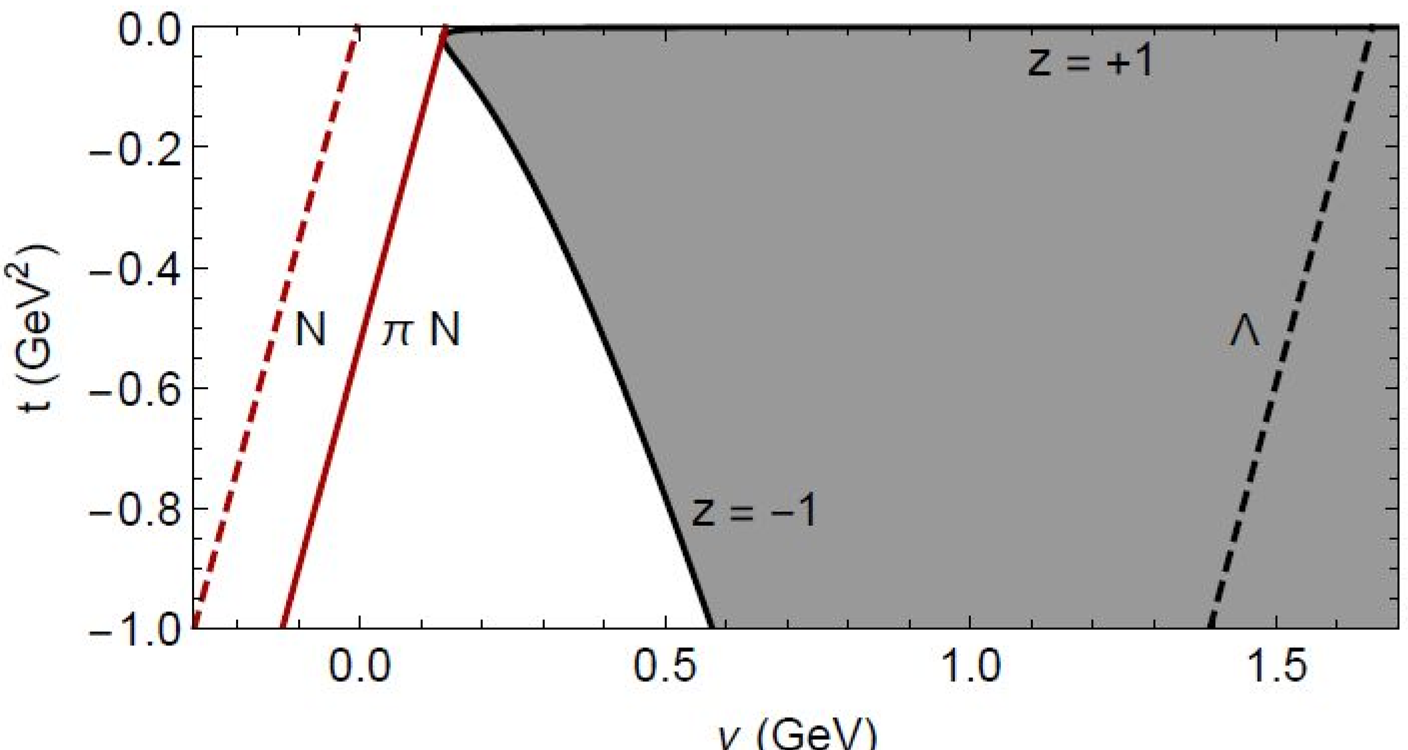
- **Finite-Energy Sum Rules** relate high and low energy regimes
  - Low-energy models provide detailed predictions for high-energy data
  - Information at the amplitude level
  - Ultimately: combined fit of low- and high-energy data

[V. Mathieu, J.N., et al. (JPAC) 1704.07684]

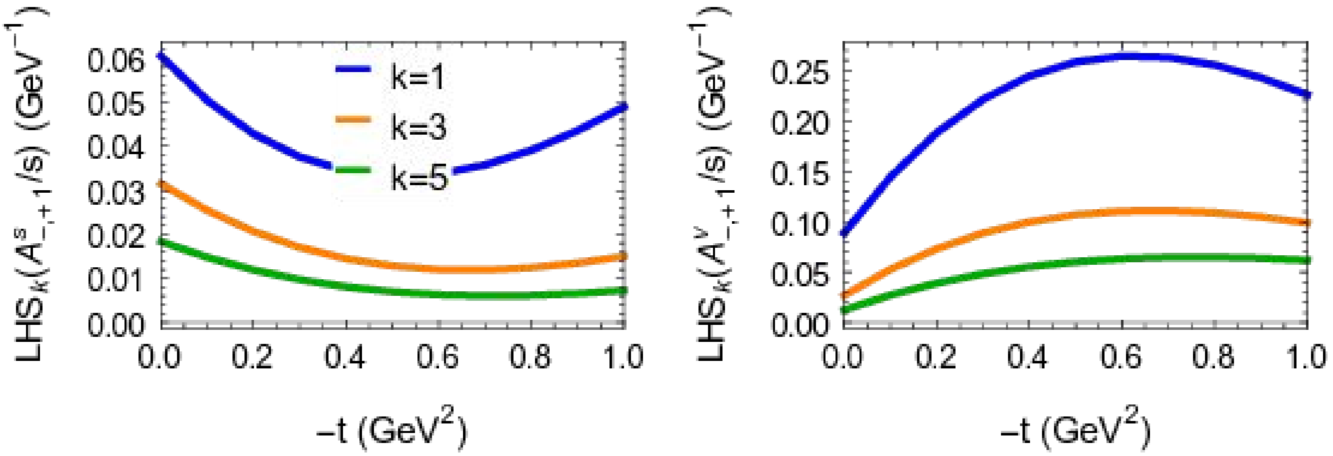
- η/η' beam asymmetry
  - Source of information about *b* and *h* radiative decays
  - Sensitive to hidden strangeness

Backup

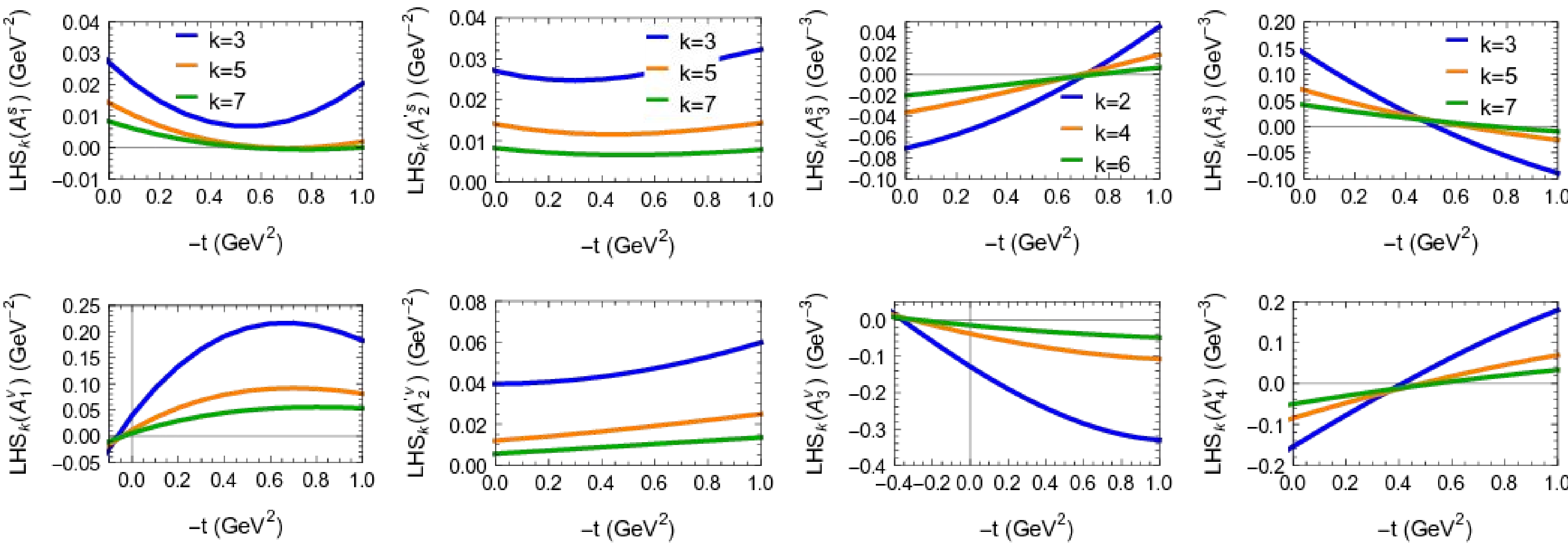
$$\alpha_{1,4}^{(\sigma)} \equiv \alpha_N(t) = 0.9(t - m_\rho^2) + 1$$
$$\alpha_{2,3}^{(\sigma)} \equiv \alpha_U(t) = 0.7(t - m_\pi^2) + 0$$

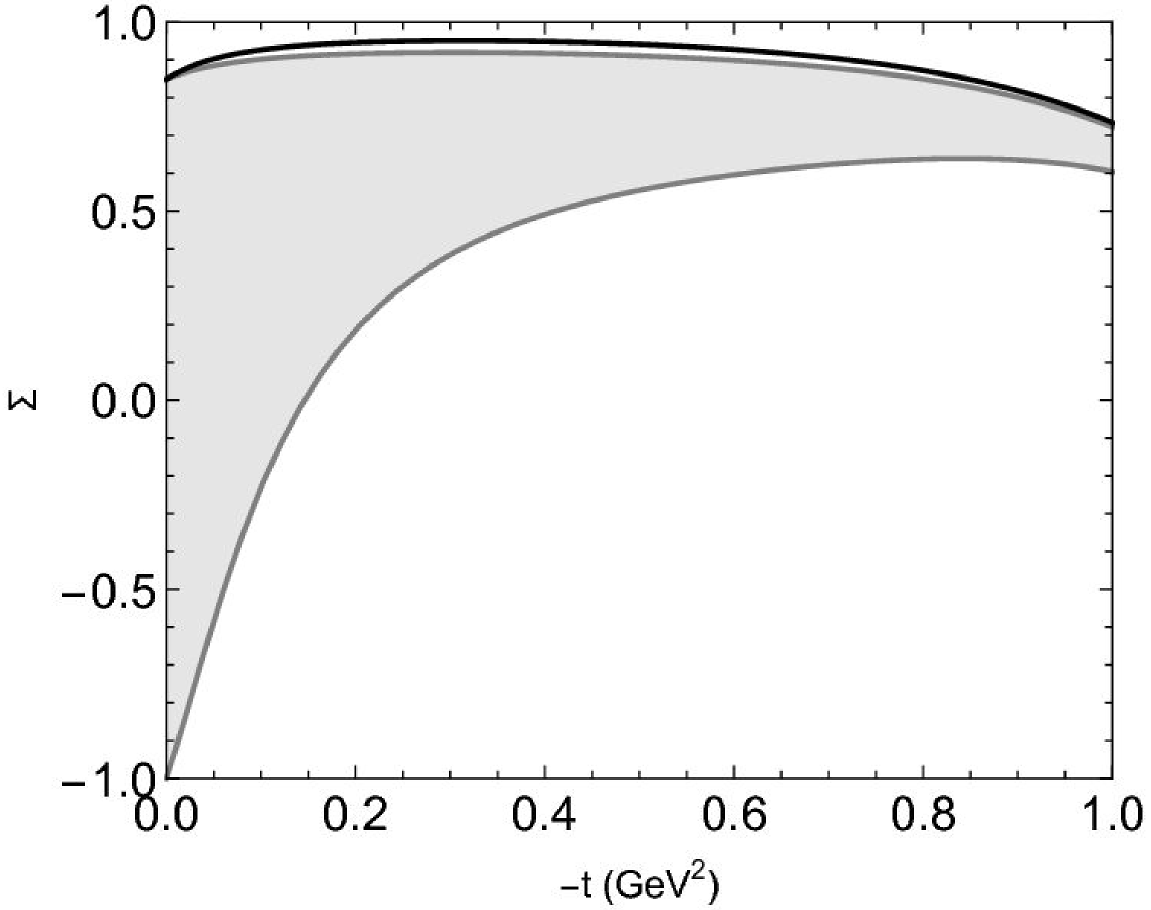


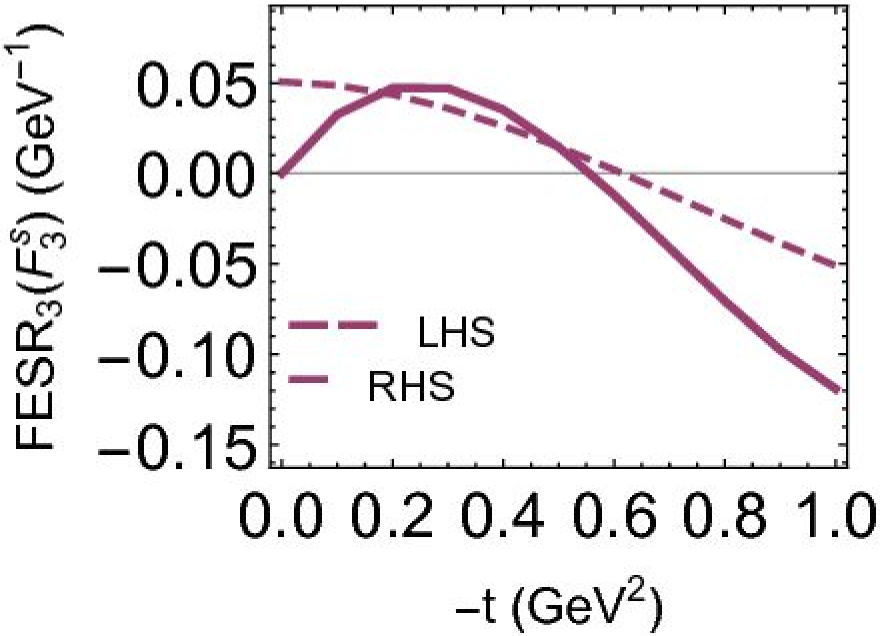
0.0
−0.2
−0.4
−0.6
−0.8
−1.0
t (GeV²)
0.0
0.5
1.0
1.5
ν (GeV)
N
π N
z = +1
z = −1
Λ

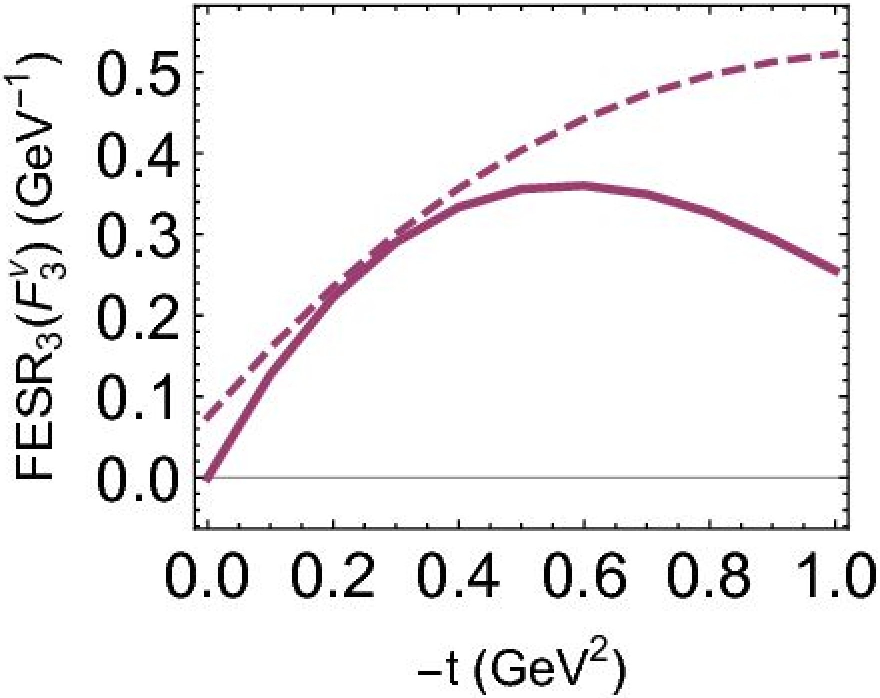


k=1
k=3
k=5
LHS$_k$($A^s_{-,+1}$/s) (GeV^{-1})
0.00
0.01
0.02
0.03
0.04
0.05
0.06
0.0
0.2
0.4
0.6
0.8
1.0
−t (GeV^2)
LHS$_k$($A^v_{-,+1}$/s) (GeV^{-1})
0.00
0.05
0.10
0.15
0.20
0.25
0.0
0.2
0.4
0.6
0.8
1.0
−t (GeV^2)

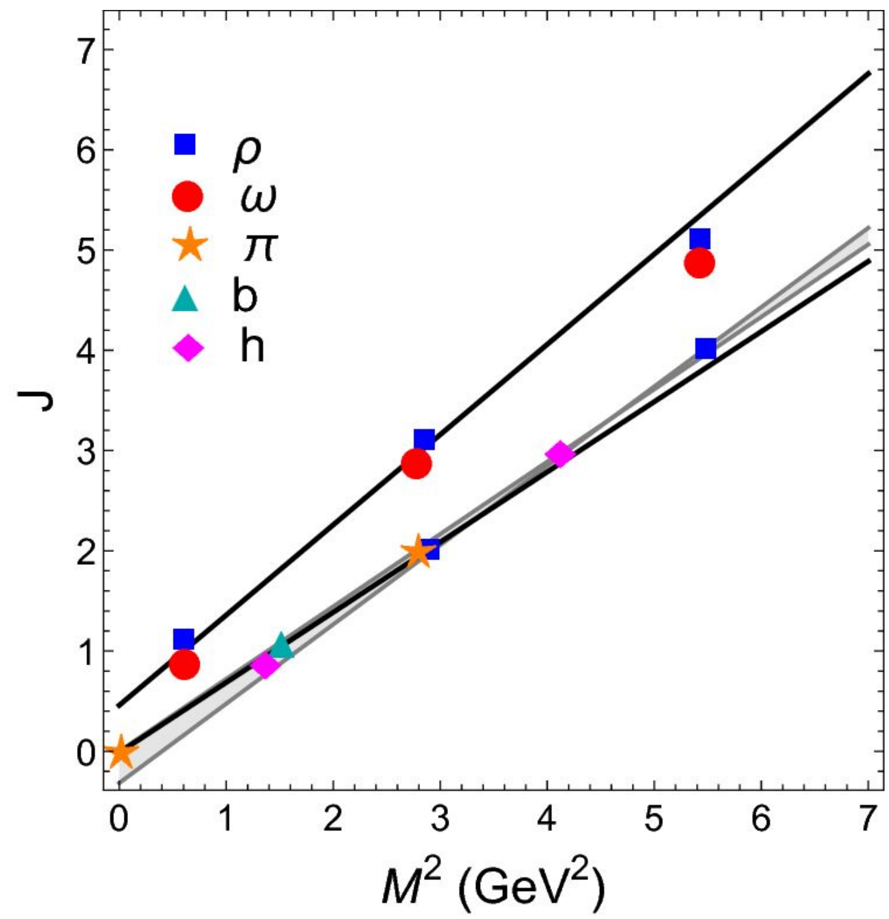

LHS$_k$(A_1^s) (GeV^{-2})
k=3
k=5
k=7
–t (GeV^2)
LHS$_k$($A_2^{'s}$) (GeV^{-2})
k=3
k=5
k=7
–t (GeV^2)
LHS$_k$(A_3^s) (GeV^{-3})
k=2
k=4
k=6
–t (GeV^2)
LHS$_k$(A_4^s) (GeV^{-3})
k=3
k=5
k=7
–t (GeV^2)
LHS$_k$(A_1^v) (GeV^{-2})
–t (GeV^2)
LHS$_k$($A_2^{'v}$) (GeV^{-2})
–t (GeV^2)
LHS$_k$(A_3^v) (GeV^{-3})
–t (GeV^2)
LHS$_k$(A_4^v) (GeV^{-3})
–t (GeV^2)


1.0
0.5
0.0
−0.5
−1.0
Σ
0.0
0.2
0.4
0.6
0.8
1.0
−t (GeV²)





$$F_3 = 2M_N A_1 - tA_4$$
$$F_4 = A_3\,.$$



ρ
ω
π
b
h
J
0
1
2
3
4
5
6
7
M^2 (GeV^2)

$$\frac{1}{\sqrt{2}s}\left(A_{+,+1}+A_{-,-1}\right)=\sqrt{-t}A_4 \quad (19)$$

$$\frac{1}{\sqrt{2}s}\left(A_{+,-1}-A_{-,+1}\right)=A_1 \quad (20)$$

$$\frac{1}{\sqrt{2}s}\left(A_{+,+1}-A_{-,-1}\right)=\sqrt{-t}A_3 \quad (21)$$

$$\frac{1}{\sqrt{2}s}\left(A_{+,-1}+A_{-,+1}\right)=-A_2'=-(A_1+tA_2) \quad (22)$$

Thus, at high energies the invariants A_3 and A_4 (A_1 and A_2') correspond to the s-channel nucleon-helicity non-flip (flip), respectively. Combining Eqs. (20) and (22) we obtain

$$A_{-,+1}=-\frac{s}{\sqrt{2}}\left(A_2'+A_1\right)\,. \quad (23)$$

$$A_{\mu_f,\mu_i\,\mu_\gamma}\underset{t\to 0}{\sim}(-t)^{n/2}\,, \quad (17)$$

where $n=|(\mu_\gamma-\mu_i)-(-\mu_f)|\geq 0$ is the net s-channel helicity flip. This is a weaker condition than the one imposed by angular-momentum conservation on factorizable Regge amplitudes,

$$A_{\mu_f,\mu_i\,\mu_\gamma}\underset{t\to 0}{\sim}(-t)^{(n+x)/2}\,, \quad (18)$$

where $n+x=|\mu_\gamma|+|\mu_i-\mu_f|\geq 1$. We summarize the